Transcript

The Math Behind Saving For College

Let's say that you've just had a child, and you expect him or her to go to college in 18 years. You at least want to have the option of being able to pay for his or her college education. You wonder as well, how much money can I start putting aside every month so that I can pay for their college education 18 years in the future?

And so I've done some research, just so we can start with some assumptions – and all of what I do in this video is all based on assumptions – and I encourage you to do the same computation, maybe with different assumptions based on what you believe is going to happen.

So I did a little bit of a research, and the current full cost of a year of a private education is approximately $45,000. A public education, the average cost is roughly half that; it's approximately $22,500.

For the sake of this video, I'll just focus on the private one, but we know in the back of our mind public is half of that. So whatever payment we come up with for private, the public, we can just do half the payment and that would be sufficient.

So how do we think about what tuition is going to be in 18 years? I think we all have a sense that it is going to be a higher. Well I did a little bit of research there and it looks like, the last few years, the full cost of a college education, the annual cost has grown by about 3% a year. So I guess we can extrapolate that forward. That's a reasonable assumption. If you think it's going to grow faster than 3%, you can put in that assumption, if you think it's going to grow slower, you can put that assumption in. But I'm just going to assume 3%,

So if we just wanted to grow this for one year by 3%, we would multiply it by 1.03. And then if we wanted to grow it for another year by 3%, we'd multiply it by another 1.03. If we wanted to do this for 18 years, we'd have to multiply it by 18 1.03s. Or there's a shorthand for that: We remember from our middle school mathematics, that's the same thing as 1.03 to the 18th power.

If we do that, let's see 45,000 x 1.03 to the 18th power, gets us to – rounding to the nearest dollar – 76,609 dollars.

I'll say approximately equal to $76,609. I have a short memory. Okay, yeah that's what it was.

So this would be one year, based on our assumptions of room and board and supplies and tuition at a private university. Now you might be tempted – oh let's just multiply this by four to get the cost of a full four-year education, but we have to remember that this inflation in cost might continue. So year 2 is going to be, we can assume 3% more. So what's that going to be?

So times 1.03 – so this is just the previous answer, grown by 3% – gets us to 78.907. So let me write these numbers. So I'm going to have – year 1 is going to be 76,609. Year 2 is going to be 78,908 round to the nearest dollar, 78,908.

So this is 78,908, and then year 3 is going to be times 1.03, is going to be 81,275.

81,275, and then year 4 – at this price you'd better hope that you child finishes in 4 years. Let's see, then we're going to grow by another 3%, times 1.03. So the senior year is going to cost 83,713.

83,713, and these are kind of eye-popping numbers, but frankly, someone who lived 18 years ago and whose child was born just 18 years ago, these were probably going to be eye-popping numbers. But regardless, if we take the sum of this, we'll get the sense of the 4-year college education.

These aren't precise numbers. These are based on our assumptions, and it is unlikely that the tuition will go up by exactly 3% every year, and once again, there's a bunch of different universities, all with different levels of tuition.

Let's figure out what this sum is. So we have 76,609 plus 78,908 plus 81,275 plus 83,713 – it gets us 320,505 dollars.

So this is 320,505 dollars for the full cost of four years at a private university, and if we assume these ratios hold and the inflation in the public is the same as the private, then the public will be roughly half of this.

So now that we know the lump sum that we need in 18 years, how much do we have to put aside each month.

Well one way to think about it is…well let's see, I'm going to put a payment – this first month of my child's life – I'm just going to do it in a new color. So I'm going to one payment, this first month of my child's life, and then the second month and then the third month, and I'm going to do this for 18 years. Eighteen years times 12 months a year that's 216 months. So I'm going to have a total of 216 payments, and they're going to have to add up to 320,505 dollars.

So another way to think about this is 216 times P is equal to 320,505, or to solve for P, you could just divide by 216: 320,505 divided by 216 is going to get you, divided by 216 is going to get you, well, almost 1,500 dollars. So 1,484 approximately.

So P is approximately 1,484. And this wouldn't be a bad approximation, but you're saying: Well look, hey I'm not just going to take these payments and stuff them into a mattress; I'm going to put them into some type of an account. Maybe I'll buy some long-term bonds that mature in time for them to be liquid when my child goes to college. Maybe I'll invest this in the stock market. At least I'm going to put it into a savings account, whatever it might be, a CD maybe. So I'm going to get some return on this. So this isn't really a fair calculation. And if you said that, you'd be absolutely right. Because on this first payment, you're going to get 216 months of interest on that. This one you're going to get 215 months, this one you're going to get 214 months of interest.

So the math gets a little bit fancier when you start trying to…to calculate that and if you're really curious about it, I encourage you to go to the Khan Academy video on Sums of Finite Geometric Series. But in those videos, I provide the formula for how to calculate this payment. This is also the way that things like mortgage payments are calculated.

So the formula we derived in those videos is that your payment, if you factor in that you're going to get some return on it – it's going to be the amount that you're saving for, so that in this case is 320,505 dollars times your monthly return that you think you're going to get.

So remember, if you're going to get, say… let's say your monthly return is say 0.3% per month…that will turn into an annual return of a little bit more than 3.6% if you compound it. If you just multiply it by 12 you get by 3.6, but you're compounding every month, so it'll be a little bit more than 3.6% per month, or 3.6% per year.

And so let's just assume that, just to make the numbers easy. So let's just assume that our monthly growth rate is 0.3%. So we're going to grow by 1.003. This is how much we're going to grow every – we're going to be 1.003 – if we start the month at 1,000 dollars, at the end of the month we're

going to have 1,003 dollars, and this once again translates into a little bit more than 3.6% return. If you think you're going to get a higher return, you'd put a higher number here. If you think you're going to get a lower return, you'd put a lower number here. But the key is, is that this is monthly. We're thinking about monthly growth. That's where that number comes from. Then you take that and you subtract 1. And so this whole expression is essentially it will give you that 0.003 or that 0.3% and then all of that over our 1.003 to the number of periods that we're compounding, that's 216 periods, 216 months, minus 1.

So we definitely need our calculator for this. This is going to give us 320,505 times – this is just going to be 0.003 – I just lose the 1, I'm subtracting 1 there. So that's our numerator and then divided by – and then we definitely need a calculator for this – 1.003 to the 216th power minus 1. And I think we deserve a drum roll right about now. We get a payment of about 1,057 dollars. Of approximately 1,057 dollars. And notice it's lower than this number because here we're assuming that each of these payments are growing. This one grows for 18 years. This one grows for 17 years and 11 months, and because of that, you can afford to put a little bit less aside based on these assumptions.

Now everything I did was for private. If you did public, it would be half this number.